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Periodical Appearance of Prime Numbers

Yoshiki Kaneoke

Department of System Neurophysiology, Graduate School, Wakayama Medical University

Highlights

1. When prime number (Pi, i=1,2,3,,,) is treated as radian, plot of (i, sin(Pi)) seems to make 20 sigmoidal lines. Exception is the 5th prime number 11, which is outside of these lines.
2. These lines seem to continue up to 99,999,999,977 investigated, and the periodicity increases as the prime number increase.
3. Three dimensional plot of (sin(Pi), cos(Pi), i) and two dimensional plot of (i, Di) (Di is the remainder after division of 180*Pi/π by 360) shows that the 20 lines are divided into two groups with a gap. Each group has 10 lines.
4. The value of angle (that is Pi radian or Di degrees) tends to cluster to make a dotted line with a specific pattern.
5. It is hypothesized that all but one (11) prime numbers are on the 20 lines and could have two different properties. It is possible, however, that several prime numbers other than 11 might be found within a gap between the two groups' lines.

Acknowledgments

1. Prime numbers used were downloaded from the Websites:
   http://www.hyogo-c.ed.jp/~meihoku-hs/club/astronomy-p.html
   http://compoasso.free.fr/primelistweb/page/prime/liste_online_en.php
2. Visualization was done with MATLAB 2015a.

First, the results for the first 2,000 (2 to 17389) prime numbers are shown in Figure 1 - 3. Second, the results are shown for the 2,000 consecutive prime numbers starting at various prime numbers. The sigmoidal lines on the plot of (i, sin(Pi)) are getting difficult to see for the larger prime numbers plots. But the plot of (i, Di) shows clear lines and gaps. The easiest way to check the fact that the dots are on the 20 lines is as follows: Print the right side of the Figure 2. Cut out the figure and make a cylinder by putting the bottom (x-axis) and the top of the figure together perfectly. The printed surface is outside, of course. It is unknown what function can draw these lines. The findings seem to be true for the several sets of the 2,000 prime numbers investigated, but not every prime numbers below 99,999,999,977 could be tested. The prime numbers used were not validated by the authorized programs.

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Figure 1. Plot of (i, sin(Pi)), i=1,2,3,, , 2000.

All dots but the dot for 11 (the plot is shown in the red circle) are on the sigmoidal lines.

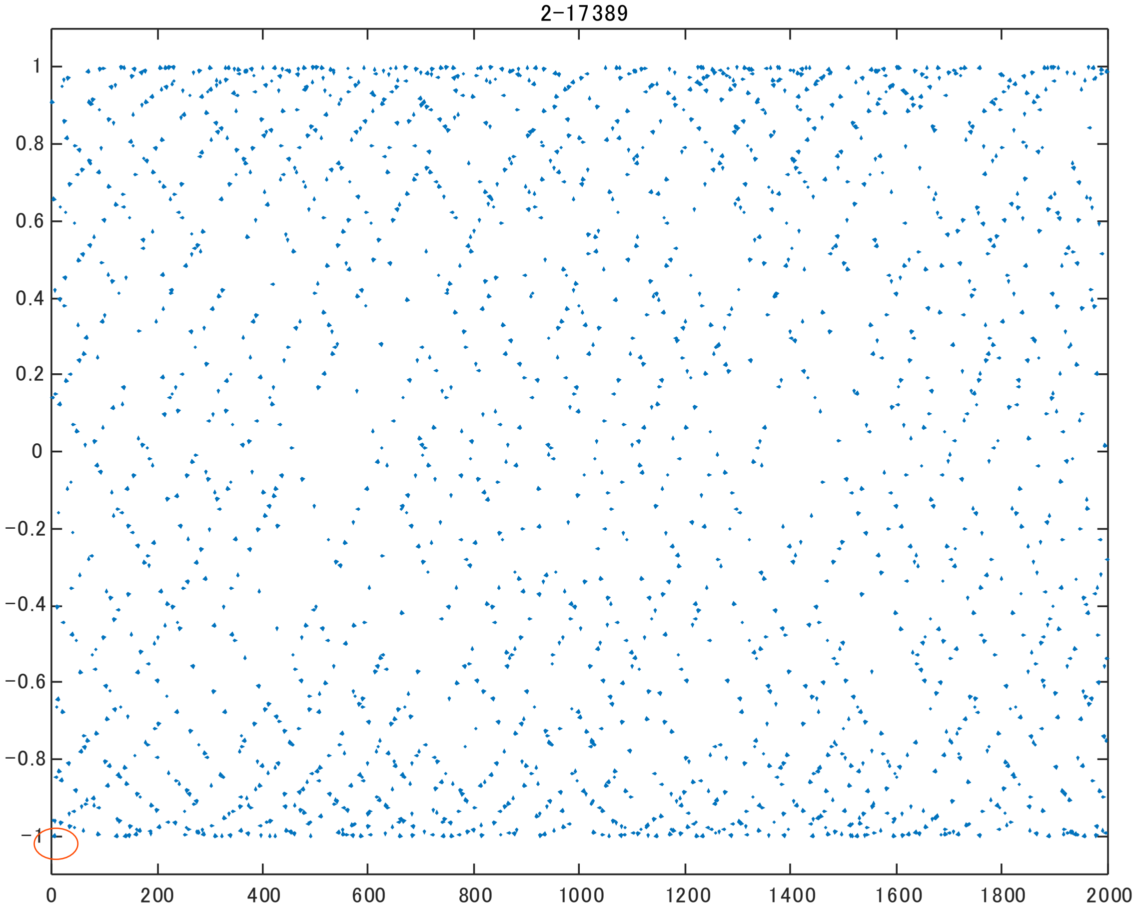


Figure 2. Three dimensional plot of (sin(Pi), cos(Pi), i) (left) and two dimensional plot of (i, Di) (right) for the first 2,000 prime numbers (2 – 17389).

The dots make obvious 20 lines except the dot for 11 shown in the red circle. Note the gap making two groups of lines, each of which consists of 10 lines.

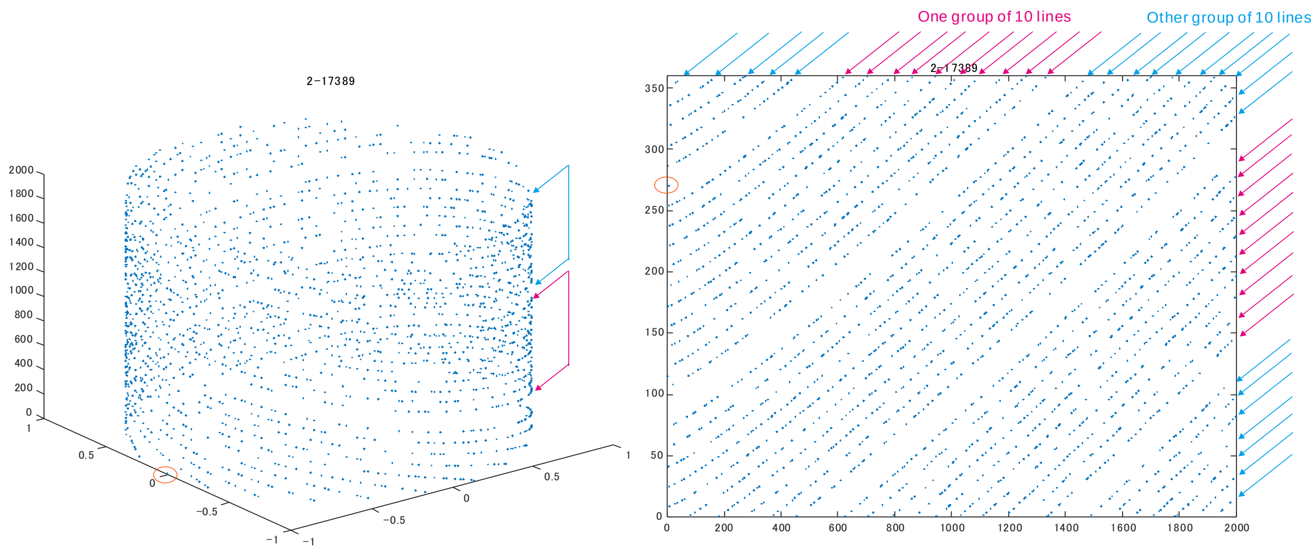


Figure 3. Three dimensional plot (sin(Pi), cos(Pi), i) shown at different angles as Fig. 2.
The angle of Pi (that is, Pi radian or Di degrees) tends to be similar as shown by the gaps (some are indicated by the red arrows). Generally, the dot line shown at the right has a specific pattern: 4 consecutive dots with a gap.

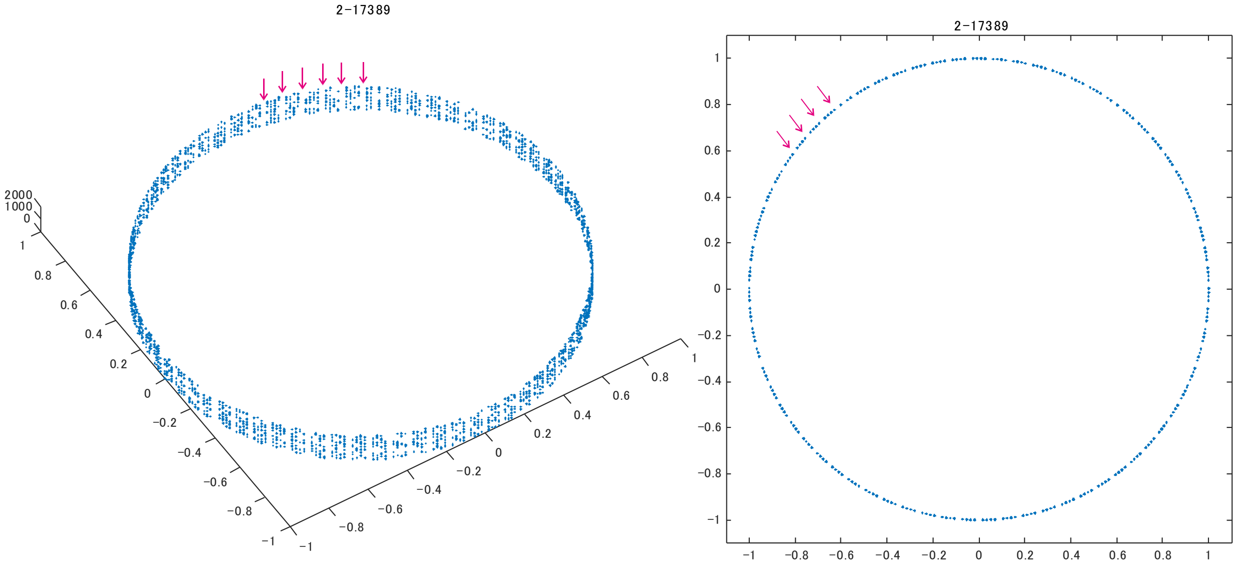


Figure 4. The same pattern is seen for the 2,000 prime numbers from 81817 to 104729.
The plots are the same as Fig. 1-3.

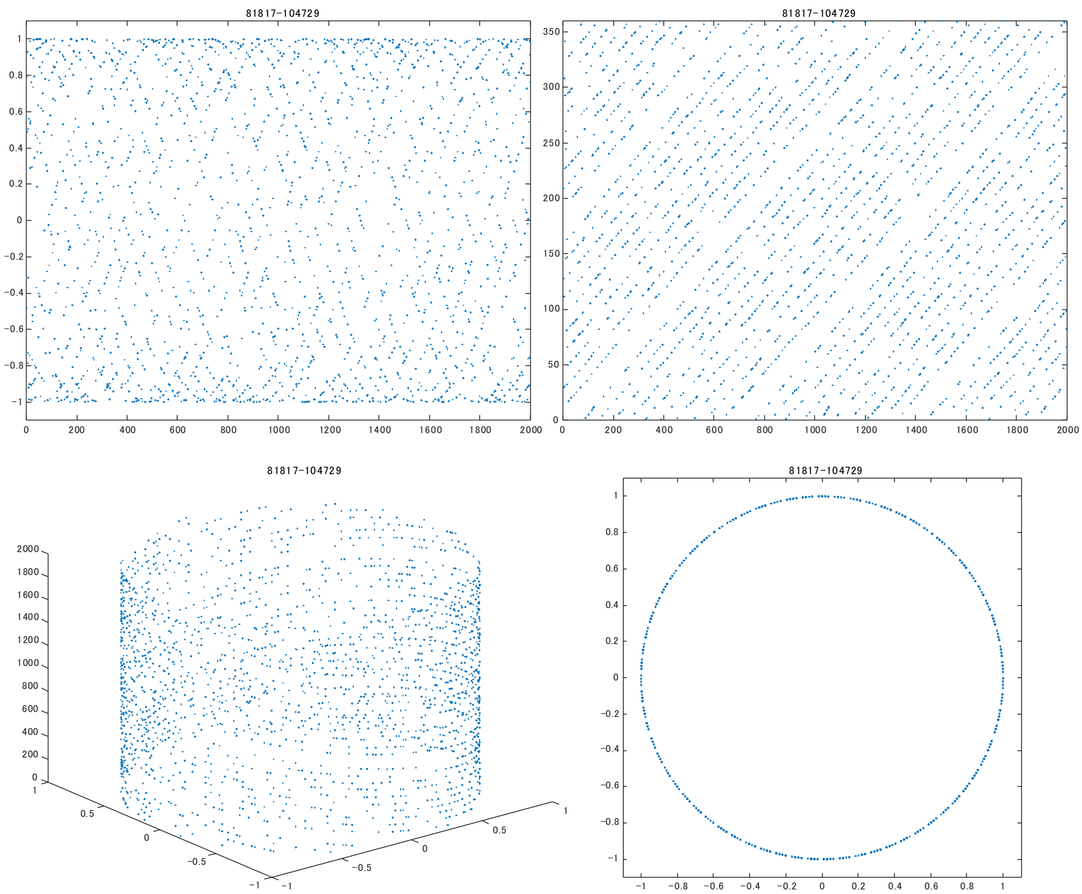


Figure 5. The 20 lines seem to continue up to 99,999,999,977.

The frequency of the sigmoidal line increases as the order increase as shown by the angle of the line (indicated by the red lines on the gaps, just to assist checking the line angle. Note that the gaps and the lines made by the dots are not straight line.). From the top, plots (i, Di) for the 2,000 prime numbers from 2 to 17,389, from 81,817 to 104,729, from 7,471,771 to 7,503,577, from 999,958,577 to 999,999,937, from 99,999,950,447 to 99,999,999,977 are shown.

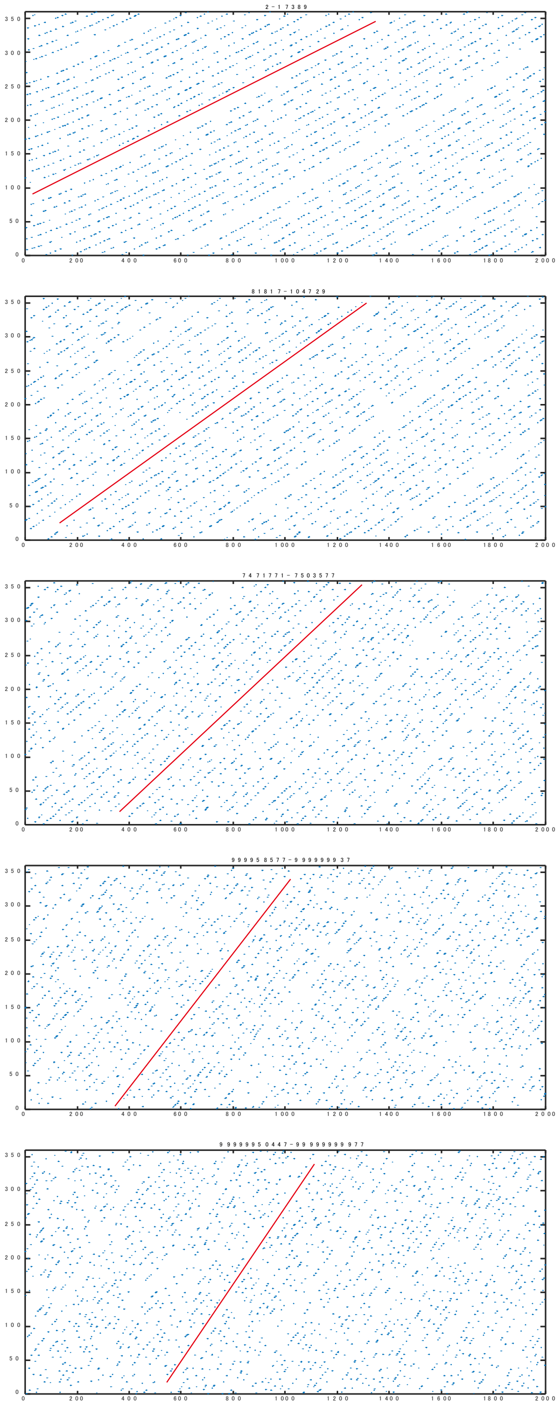


Figure 6. The specific dots pattern is also seen up to 99,999,999,977 investigated.
The pattern means that the prime number angle (Di) tends to gather within restricted small ranges when checked with a small consecutive number of prime numbers like 2,000.

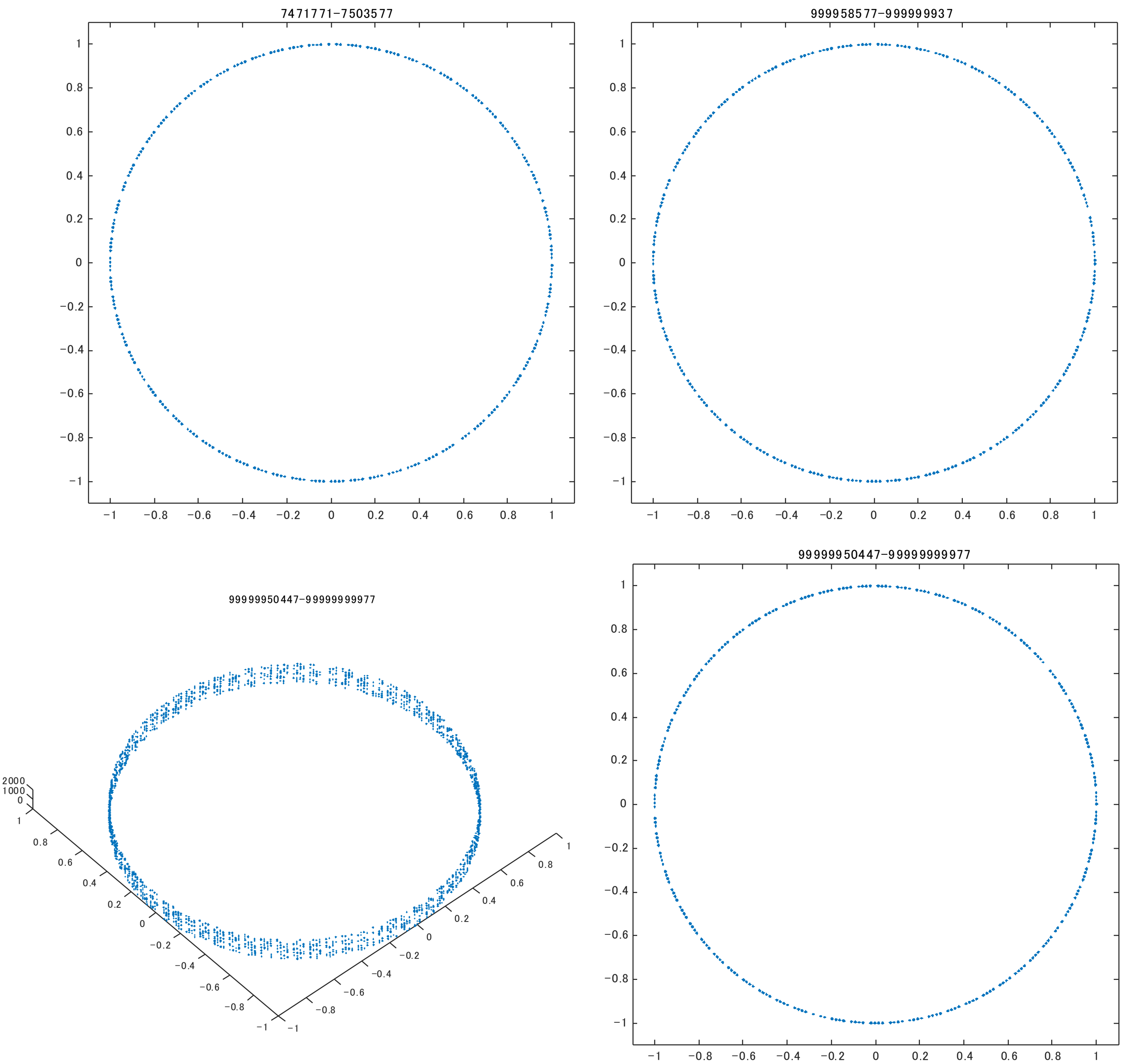
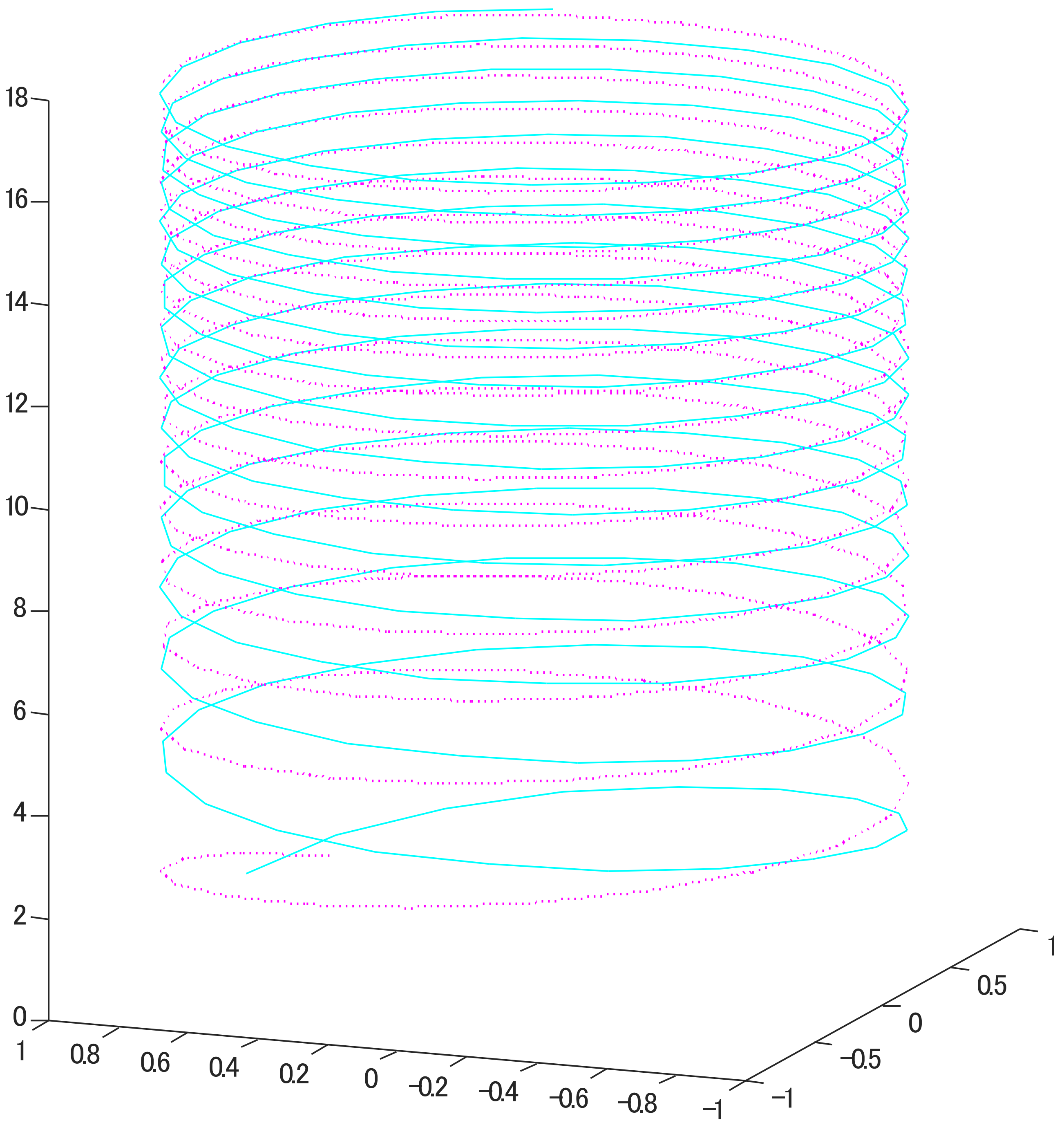


Figure 7. Model of the prime numbers periodicity.

Radian angle of the prime number seems to be on the 20 lines, which could be divided into two groups (illustrated below by the cyan and the magenta lines). Each group has 10 lines and is separated by the gap of one line. One prime number 11 is found in the gap. These lines continue increasing the periodicity. As shown below, the space between the lines are getting smaller as the prime number increases.



Appendix

Matlab code for the plots.

```
%%
P=primes(17390);  %first 2000 prime numbers
I=1:2000;
D=2000;
figure
plot3(sin(P(D-1999:D)),cos(P(D-1999:D)),I,'.')
title([num2str(P(D-1999)),'-',num2str(P(D))])

figure
plot(sin(P(D-1999:D)),cos(P(D-1999:D)),'.')
title([num2str(P(D-1999)),'-',num2str(P(D))])
axis([-1.1 1.1 -1.1 1.1])
axis square

figure
plot(sin(P(D-1999:D)),'.')
title([num2str(P(D-1999)),'-',num2str(P(D))])
axis([0 2001 -1.1 1.1])

figure
plot(mod(360*P(D-1999:D)/(2*pi),360),'.')
title([num2str(P(D-1999)),'-',num2str(P(D))])
axis([0 2001 0 360])
%%
```